Risk taking behavior in the presence of nonconvex asset dynamics

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ABSTRACT

A growing literature on poverty traps emphasizes the links between multiple equilibria and risk avoidance. However, multiple equilibria may also foster risk taking behavior by some poor people. We illustrate this idea with a simple analytical model in which people with different wealth and ability endowments make investment and risky activity choices in the presence of known nonconvex asset dynamics. This model underscores a crucial distinction between familiar static concepts of risk aversion and forward-looking dynamic risk responses to nonconvex asset dynamics. Even when unobservable preferences exhibit decreasing absolute risk aversion, observed behavior may suggest that risk aversion actually *increases* with wealth near perceived dynamic asset thresholds. Although high ability individuals are not immune from poverty traps, they can leverage their capital endowments more effectively than lower ability types and are therefore less likely to take seemingly excessive risks.

JEL Codes: D81, O12, D90

Introduction

A growing literature on poverty traps and multiple equilibria associated with nonconvex asset dynamics emphasizes the relationship between risk avoidance and poverty (Bardhan, Bowles and Gintis 2000; Carter and Barrett 2006; Zimmerman and Carter 2003). There exist at least two distinct relationships noted to date. First, if agent preferences exhibit decreasing absolute risk aversion – a very common assumption – then people who start poor should choose lower risk, lower expected return portfolios that may leave them poorer in long-run equilibrium than those who begin with greater wealth (Bardhan, Bowles and Gintis 2000). In this view, initial endowments combine with risk to generate multiple equilibria. Some empirical studies support this hypothesis (e.g., Carter 1997; Rosenzweig and Binswanger 1993). Second, a more recent literature suggests that nonconvex asset dynamics characteristic of multiple equilibrial systems may also create incentives to smooth assets, rather than consumption, for those at or just above the threshold at which wealth dynamics bifurcate (Barrett et al. 2006; Carter et al. 2007; Hoddinott 2006; McPeak 2004; Zimmerman and Carter 2003). The intuition behind such asset smoothing is simple: people safeguard the productive assets on which their future livelihood depends if liquidating assets so as to smooth consumption pushes them below a threshold at which they expect exogenous asset dynamics to suddenly cause further asset loss. The difference between these two views has important implications for behavior under risk. Whereas the first sees causation running from risk preferences to wealth dynamics, the second suggests it might run from wealth dynamics to risk preferences as manifest in risk-taking behaviors. This paper develops this latter, largely overlooked point.

By suggesting that risk-taking behavior might be shaped by wealth dynamics, the asset smoothing hypothesis raises the intriguing, complementary possibility that multiple equilibria associated with nonconvex asset dynamics could lead to seemingly excessive risk taking behavior among those already below a dynamic asset threshold. Such individuals might be driven by desperation to take chances when a safe strategy is unlikely to break them out of a poverty trap. This observation is not exactly new, but explanations for this behavior have previously relied on unconventional preferences, typically with embedded latent wealth or asset dynamics. Most famously, Friedman and Savage (1948) motivated their double inflection, "wiggly" utility curve with a loose reference to implicit wealth dynamics that make it difficult for individuals to move to higher socioeconomic classes and hence risk seeking when upside payoffs allow them to move to a higher class. In their words, "the segments of diminishing marginal utility correspond to socioeconomic classes, the segment of increasing marginal utility to a transitional stage between a lower and a higher socioeconomic class" (p.304). Others subsequently explored these underlying dynamics slightly more explicitly, but continued to embed them in preferences. Many of these earlier models foreshadow some of the key features of the multiple equilibria models of the 1990s, namely, indivisible human capital investments (Yew Kwang 1965), credit market imperfections (Hakansson 1970; Masson 1972), and nutritional subsistence constraints (Kunreuther and Wright 1979; Masson 1974; Roumasset 1976). With the benefit of recent work on poverty dynamics, Banerjee (2004) addresses asset dynamics more explicitly by contrasting poverty above dynamic asset thresholds (vulnerability) with poverty below these thresholds (desperation), a characterization we build on in this paper.

This paper considers how the existence of thresholds in the laws of motion describing asset dynamics in multiple equilibrial systems might induce extraordinary risk taking by certain subpopulations among the poor and how risky behavior might vary according to latent ability endowments. We illustrate these ideas with a simple analytical model in which people make activity and investment choices. One activity involves timeless risk with zero expected return. Investment inherently trades lower current consumption for higher future consumption. While no risk averse agent would engage in the risky activity under standard assumptions, we show that risk taking occurs, and is nonmonotonically related to liquid wealth within a key asset range, when nonconvex asset dynamics characterize the system. Indeed, among those who take on seemingly excessive risk, wealth and risk taking are inversely related even when preferences exhibit decreasing absolute risk aversion. The core intuition is that some people will take chances so as to avoid predictable collapse if not taking chances leads them deeper into a trap. Because actual risk-taking behavior in observational data responds to the underlying asset dynamics agents perceive in a system, this analysis carries significant implications for the estimation of risk preferences, especially the assumption of a monotone relation between risk premia and wealth.

The Model

Suppose individuals have a strictly concave, contemporaneous utility function defined over consumption $c\geq 1$ as $u(c)=\ln(c)$ and face two periods of decision making. Intertemporally additive utility for these two periods is given by $U(c_1,c_2)=u(c_1)+\delta u(c_2)$, where $\delta<1$ is a discount factor. Individuals have three initial endowments. W is unproductive liquid wealth (including food) which can be stored or consumed but does not generate any flow of real income. H_1 is illiquid, productive wealth (e.g., human capital) which generates income without depreciation provided that consumption level is also maintained. Finally, individuals are endowed with ability $r_i \sim U\left[r_L, r_H\right]$, which represents a non-risky return on H each period. Given these wealth and ability endowments, individuals have three choice variables: c_t is consumption,

K represents investment of W in future productive wealth such that $H_2 = H_1 + sK$ with s>0, and Y is the allocation of W to a fair (i.e., zero expected value) coin toss gamble that pays $2Y$ in period t=1 with probability $p = 1/2$ and 0 otherwise.

Without loss of generality, we capture wealth dynamics in this simple model with a single, stark consumption threshold, $\bar{c}$, and bound consumption from below at a unit. Specifically, we assume that (a) if $c_1 < \bar{c}$, then $H_2 = r_i^{-1}$ (instead of $H_2 = H_1 + sK$) so that $r_i H_2 = 1$ and (b) $r_L > (H_1 + sK)^{-1}$ so that $r_i H_2 > 1 \;\; \forall i$ if $c_1 \geq \bar{c}$. These assumptions ensure that $u\left(r_i H_2 \middle| c_1 < \bar{c}\right) = 0 < u\left(r_i H_2 \middle| c_1 \geq \bar{c}\right)$, implying that insufficient consumption can be understood as permanent disability (in the limit, death). Lastly, we assume for simplicity that individuals choose K and Y and observe the outcome of the coin toss before choosing c_1. The expected utility model implied by this set up is:

$$
\begin{aligned}
&\max_{\substack{Y\in[0,W]\\K\in[0,W]\\c_t\geq 1}} E\left[U(c_1,c_2)\right] = E\left[\ln(c_1) + \delta \ln(c_2)\right] \\
&s.t. \qquad Y + K \leq W \\
&\qquad\qquad c_1 \leq W + \tilde{y} + r_i H_1 - K \\
&\qquad\qquad c_2 \leq W + \tilde{y} + r_i H_1 - K - c_1 + r_i H_2 \\
&\qquad\qquad H_2 = \begin{cases} H_1 + sK & if\ c_1 \geq \bar{c} \\ r_i^{-1} & if\ c_1 < \bar{c} \end{cases} \\
&\qquad\qquad \tilde{y} = \begin{cases} 2Y, & p = 1/2 \\ 0, & 1-p = 1/2 \end{cases}
\end{aligned}
\tag{1}
$$

The threshold $\bar{c}$ creates an important nonconvexity in asset dynamics, as the discontinuous law of motion for productive wealth generates multiple dynamic equilibria, with a lower stable dynamic equilibrium at H=r_i^{-1}. This simple dynamic asset threshold effectively links periods 1 and 2 in such a way that for some parameter values there is a stark divergence between standard static risk preferences, as reflected in the (unobservable) Arrow-Pratt coefficient of absolute risk aversion ($-u''/u'$), and what might be termed a "dynamic risk response" as reflected in observed risk-taking behavior given agent knowledge of the prevailing asset dynamics of the system. That is, some individuals will risk a portion of their wealth in a way that seems contrary to their risk averse preferences in an attempt to survive until period 2.

To solve the model, consider three cohorts of individuals: (A) a hopelessly trapped cohort, (B) a desperate cohort of individuals for whom the gamble is their only hope for survival, and (C) a richer cohort that is safely above the consumption threshold. Individuals in all three cohorts are Arrow-Pratt risk averse, as the static coefficient of absolute risk aversion, $SARA \equiv u''/u' = 1/c = 1/(W+r_iH_i) > 0$, given our logarithmic utility function specification.[1] Because the timeless gamble has an expected payoff of zero, they will choose $Y^*>0$ if and only if winning the gamble brings some benefit in addition to the direct monetary value of the win – namely, preservation of productive assets into period 2. This is only true for individuals whose initial endowments satisfy the following two conditions: (i) $W + r_i H_1 < \overline{c}$ and (ii) $2W + r_i H_1 \geq \overline{c}$. The first condition ensures that individual *i*'s consumption will be insufficient *for certain* if he ignores the coin toss opportunity; the second ensures that a bet of $Y=W$ (or less if (ii) holds with strict inequality) will provide a $1/n>0$ chance of reaching the consumption threshold, $\overline{c}$. Together, these two conditions define cohort B as individuals for whom the coin toss is their only chance to escape otherwise-certain penury. Initial endowments in the trapped cohort A satisfy condition (i) but not (ii); even an "all or nothing" bet of W offers no hope of preserving H_1. Initial endowments in the richer cohort C satisfy condition (ii) but not (i) such that asset retention is guaranteed. Finally, note that these cohort conditions are defined by wealth but also conditioned on ability.

[1] In a static setting, $Y^*=K^*=0$ at any optimum, thus $c^*=w+r_iH_i$ by the binding budget constraint.

Cohorts A, B and C in this model are depicted graphically in H_1 and W space in figure 1. To capture the effect of ability on risk taking behavior, we depict the boundaries of behavior for each cohort as mapped out by the highest ability (r_H) and lowest ability (r_L) individuals in these cohorts. The outlined triangles depict cohort B's initial endowment range in H_1 and W space: the smaller, cross-hatched triangle represents highest ability individuals, while the shaded triangle represents lowest ability individuals. Cohort A (cohort C) encompasses highest and lowest ability individuals with initial endowments southwest (northeast) of these respective triangles. The boundary between cohorts A and B in figure 1 is a function not just of initial asset and ability endowments but also of the odds offered on the gamble. Gambles with worse than 50:50 odds would offer a possible escape route from long-term poverty to those who otherwise face certain asset loss, creating profit-taking opportunities for those who offer such gambles to the poor. In particular, if the gamble paid nY with probability $1/n$ and 0 otherwise, the lower left corner of the cohort B triangles would shift leftward as n>2 increases. While the distinctly "do-or-die" flavor of this simple two period model exaggerates this desperate risk taking effect, similar skewness seeking behavior is often observed in lotteries (Yew Kwang 1965) or horse track betting (Golec and Tamarkin 1998).

The solution of this model for cohorts A and C is straightforward. The coin toss gamble offers nothing in addition to the direct monetary gain or loss and is therefore unappealing to both, so $Y^{*A} = Y^{*C} = 0$. Cohort A will not reap any return on investment and hence has no incentive to invest, so $K^{*A}=0$. Cohort C, on the other hand, has a clear incentive to invest provided s and r_i are sufficiently high. In particular, individuals in this cohort face the following simplified problem

(2) $$\max_{K \in [0,W]} U = \ln\left(W + r_i H_1 - K\right) + \delta \ln\left(r_i (H_1 + sK)\right)$$

with necessary first order condition and K^{*C} given by

(3)
$$\frac{\partial U}{\partial K}=\frac{-1}{W+r_iH_1-K}+\frac{\delta r_i s}{r_i(H_1+sK)}=0$$
$$K^{*C}=\frac{1}{1+\delta}\left[\delta W+\left(\delta r_i-s^{-1}\right)H_1\right]$$

Since individuals in the desperate cohort B are still contemporaneously risk averse, they will only risk the minimum amount required to get them to $\overline{c}$ as determined by the distance between their current wealth and the threshold, adjusted for any investments in *K*.[2] Thus, for this cohort $Y^{*B}=\overline{c}-(W+r_iH_1-K)$ and the model becomes

(4)
$$\max_{K\in[0,W]} EU=0.5\ln\left(\overline{c}\right)+0.5\ln\left(2\left(W+r_iH_1-K\right)-\overline{c}\right)+\delta\left[0.5\ln\left(r_i\left(H_1+sK\right)\right)\right]$$

with the solution for K^{*B} given by

(5)
$$\frac{\partial E}{\partial K}=\frac{-1}{\left(2\left(W+r_iH_1-K\right)-\overline{c}\right)}+\frac{0.5\delta r_i s}{r_i\left(H_1+sK\right)}=0$$
$$K^{*B}=\frac{1}{1+\delta}\left[\delta W+\left(\delta r_i-s^{-1}\right)H_1-0.5\delta\overline{c}\right]=K^{*C}-\frac{0.5\delta\overline{c}}{1+\delta}$$

Optimal investment for the poorest individuals in cohort B, for whom $2W+r_iH_1=\overline{c}$, is therefore

(6)
$$K^{*B}\Big|_{2W+r_iH_1=\overline{c}}=\frac{H_1}{1+\delta}\left(1.5\delta r_i-s^{-1}\right)$$

which marks the lower bound on investment for cohort B since K^{*B} is increasing in W, H_1 and r_i. Thus, $K^{*B}>0$ for all individuals in this desperate cohort for whom $r_i>\left(1.5\delta s\right)^{-1}$. Investment levels for cohort B are lower than for cohort C because the threshold presents a relevant threat to asset preservation, which reduces the marginal value of investing by the 0.5 probability of not surviving to reap a return. While optimal investment is monotonically increasing in both *W* and

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[2] This is true as long as the discount factor δ is not so small that the present value of a positive utility in period two is trivial, which we assume throughout.

H_1 for both cohorts, moving from B to C across the boundary $W + r_i H_1 = \overline{c}$ entails a discrete jump in optimal investment K^*. Figure 2 depicts the weakly monotone, discontinuous optimal investment schedule in W and H_1 space assuming $r_L = (1.5\delta s)^{-1}$ and $r_H > (1+\delta)^{-1} + \delta s^{-1}$.

We can now compare two measures of risk aversion across the wealth distribution: the standard *static* coefficient of absolute risk aversion, $SARA = 1/(W+r_iH_i)$, and a measure of *dynamic* risk aversion defined as $DRA \equiv -Y^*/W$. These measures are comparable in sign since risk seeking behavior implies that both are negative. In figure 3, we depict *SARA* and *DRA* for the highest and lowest ability types with $H_1 = \overline{H}_{1H}$. Over the asset ranges corresponding to cohorts A and C, *DRA*=0 and *SARA*>0, and there is no dynamic risk response. But the presence and perception of nonconvex asset dynamics drive a wedge between static and dynamic risk aversion such that *SARA*>0 and *DRA*<0. Furthermore, these dynamics generate an observable behavioral response that suggests a locally inverse relationship between wealth and risk taking even though unobservable static risk preferences require the opposite. Finally, this model demonstrates the mitigating effect of ability on this desperate, dynamic risk response: high ability individuals exhibit dynamic risk taking over a lower and narrower range of wealth than do low ability individuals.

Discussion

The observation that perceived dynamic asset thresholds can invoke risk responses dates at least to Friedman and Savage's (1948) classic article on risk and wealth, but this paper is the first to bring these dynamics out of the black box of preferences. Making nonconvex asset dynamics explicit in models of decision making under risk draws a helpful distinction between static risk preferences and dynamic risk responses.

Our model requires that individuals accurately perceive the location and severity of a critical dynamic threshold. This is an obvious necessary condition to any behavioral risk response to asset dynamics. The more precisely people perceive the dynamics of asset accumulation, the sharper the distinction between static and dynamic risk responses. Indeed, if people could perceive these dynamics perfectly – admittedly an extreme and unlikely case – two separate and relevant types of risk would emerge across all wealth levels: (1) *static prospect risk* associated with changes in wealth and (2) *dynamic inertia risk* associated with the forces on absolute wealth exerted by persistent, underlying dynamics. One could in principle decompose observed behavior into these components. Indeed, Chevalier and Ellison (1997) use data on mutual fund portfolios to do this empirically and find that fund managers tend to gamble with riskier fourth quarter portfolios in order to catch the market or make "best fund" lists because of nonlinear fund size dynamics. Surely such systematic and strategic tradeoffs between static prospect risks and dynamic inertia risks are not confined to Wall Street alone and could be problematic in any empirical application that takes the standard static risk preference approach.

While it is unrealistic to expect individuals to perfectly perceive nonconvex asset dynamics that are far more subtle and complex than the stark threshold in this simple model, a growing body of empirical evidence suggests that in at least some contexts people indeed accurately perceive the location of critical thresholds in asset space. For example, Hoddinott (2006) finds that Zimbabwean households clearly behave as if a pair of oxen represents an asset threshold below which they strive not to fall. Santos and Barrett (2006), meanwhile, show that Ethiopian pastoralists' subjective expectations of herd transitions conditional on rainfall realizations yield unconditional asset dynamics expectations virtually identical to those observed in separate herd history data from the same region (Lybbert et al. 2004). These studies suggest that people are more likely to perceive thresholds that occur at a clear discontinuity in asset space and that have severe, identifiable consequences. Contexts with nonconvex wealth dynamics, simple and discrete asset spaces, and discernible path dynamics with seasonal or annual – as opposed to daily or weekly – cash flows may be especially likely to evoke a dynamic risk response of the sort we model.

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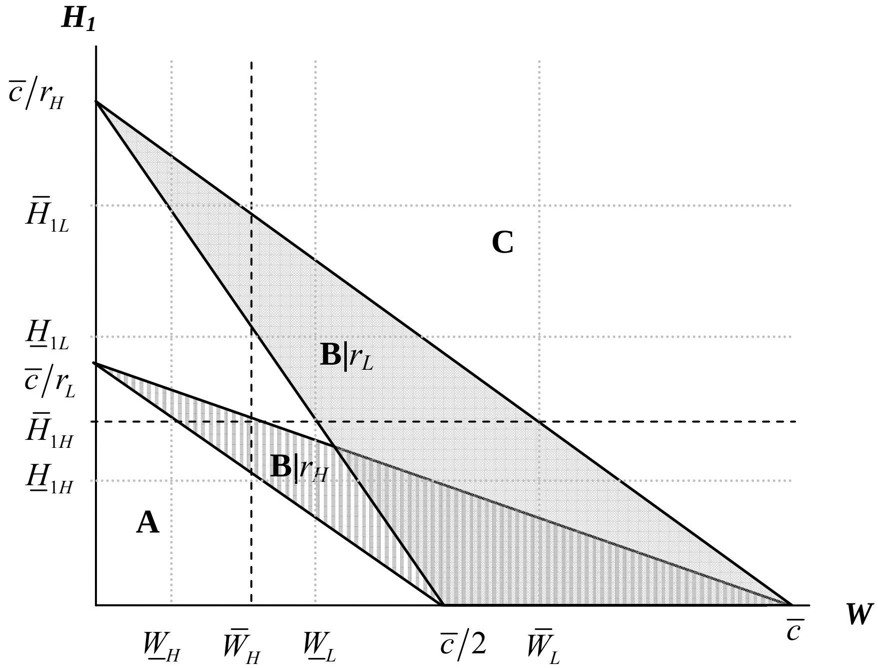

Figure 1 Graphical depiction of cohorts A, B and C in asset space for highest and lowest ability types

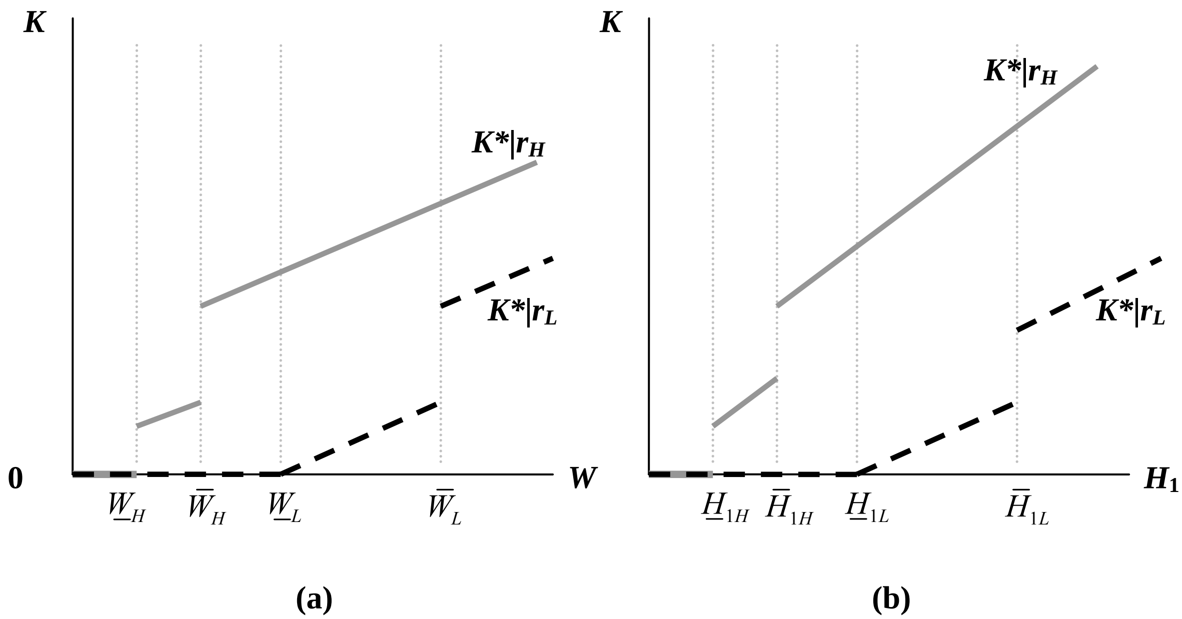

Figure 2 Optimal human capital investment K^* for highest and lowest ability types at (a) $\overline{H}_{1H}$ and (b) $\overline{W}_H$ from figure 1

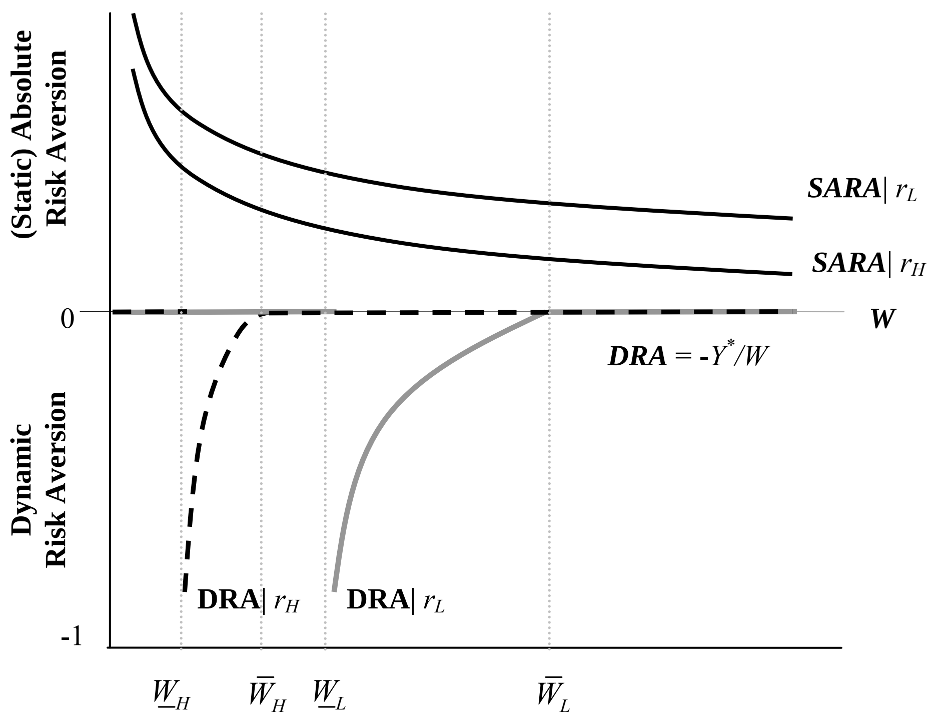

Figure 3 Absolute risk aversion and dynamic risk aversion for highest and lowest ability types at $\overline{H}_{1H}$ from figure 1